5-2

5-2 RATIOS AND PROPORTIONS

1 Write and Use Ratios

A **ratio** is a comparison of two quantities using division. The ratio of quantities a and b can be expressed as a to b, $a:b$, or $\frac{a}{b}$, where $b \neq 0$. Ratios are usually expressed in simplest form.

↳ read "a to b"

The aspect ratios 32:18 and 16:9 are equivalent.

$$\frac{\text{width of screen}}{\text{height of screen}} = \frac{32 \text{ in.}}{18 \text{ in.}} \quad \text{Divide out units.}$$

$$= \frac{32 \div 2}{18 \div 2} \text{ or } \frac{16}{9} \quad \text{Divide out common factors.}$$

Example 1:

SPORTS A baseball player's batting average is the ratio of the number of base hits to the number of at-bats, not including walks. Minnesota Twins' Joe Mauer had the highest batting average in Major League Baseball in 2006. If he had 521 official at-bats and 181 hits, find his batting average.

Divide the number of hits by the number of at-bats.

$$\frac{\text{hits}}{\text{at-bats}} \quad \frac{181}{521} \approx \frac{.347}{1} \approx .347$$

$\boxed{\text{Mauer's batting average was .347.}}$

Example 2:

In Logan's high school, there are 190 teachers and 2650 students. What is the approximate student-teacher ratio at his school?

$$\frac{\text{student}}{\text{teacher}} \quad \frac{2650}{190} \approx \frac{13.9}{1} \approx \frac{14}{1}$$

$\boxed{\text{The student-teacher ratio is approximately 14 students per teacher.}}$

5-2

Extended ratios can be used to compare three or more quantities. The expression $a:b:c$ means that the ratio of the first two quantities is $a:b$, the ratio of the last two quantities is $b:c$, and the ratio of the first and last quantities is $a:c$.

Example 3: hint: put an x with each term of the ratio.

The ratio of the measures of the angles in a triangle is 3:4:5. Find the measures of the angles.

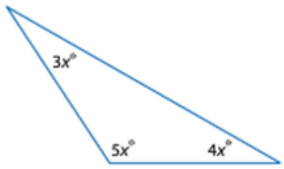
↳ $3x, 4x, 5x$

triangle sum = 180

$3x + 4x + 5x = 180$

$\frac{12x}{12} = \frac{180}{12}$

$x = 15$



$3x = 3(15) = 45$

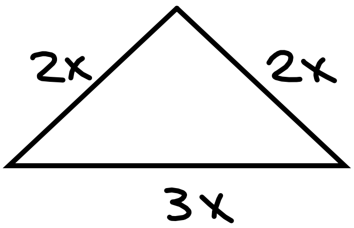
$4x = 4(15) = 60$

$5x = 5(15) = 75$

Example 4:

In a triangle, the ratio of the measures of the sides is 2 : 2 : 3 and the perimeter is 392 inches. Find the length of the longest side of the triangle.

$2x, 2x, 3x$



P = side 1 + side 2 + side 3

$392 = 2x + 2x + 3x$

$\frac{392}{7} = \frac{7x}{7}$

$56 = x$

$3x = 3(56) = 168$

$2x = 2(56) = 112$

$2x = 2(56) = 112$

5-2

2 Use Properties of Proportions

An equation stating that two ratios are equal is called a **proportion**. In the proportion $\frac{a}{b} = \frac{c}{d}$, the numbers a and d are called the **extremes** of the proportion, while the numbers b and c are called the **means** of the proportion.

$$\text{extreme} \rightarrow a \quad c \leftarrow \text{mean}$$
$$\frac{a}{b} = \frac{c}{d}$$
$$\text{mean} \rightarrow b \quad d \leftarrow \text{extreme}$$

The product of the extremes ad and the product of the means bc are called **cross products**.

KeyConcept Cross Products Property

Words	In a proportion, the product of the extremes equals the product of the means.
Symbols	If $\frac{a}{b} = \frac{c}{d}$ when $b \neq 0$ and $d \neq 0$, then $ad = bc$.
Example	If $\frac{4}{10} = \frac{6}{15}$, then $4 \cdot 15 = 10 \cdot 6$. $60 = 60$

You will prove the Cross Products Property in Exercise 41.

The converse of the Cross Products Property is also true. If $ad = bc$ and $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} = \frac{c}{d}$. That is, $\frac{a}{b}$ and $\frac{c}{d}$ form a proportion. You can use the Cross Products Property to solve a proportion.

Example 5: Solve each proportion.

5a. $\frac{-4}{7} = \frac{6}{(2y-5)}$

$-4(2y-5) = 7(6)$

$-8y + 20 = 42$

$-20 \quad -20$

$-8y = 22$

$\frac{-8y}{-8} = \frac{22}{-8}$

$y = -2.75$

5b. $\frac{7}{(z-1)} = \frac{9}{(z+4)}$

$7(z+4) = 9(z-1)$

$7z + 28 = 9z - 9$

$-7z + 9 \quad -7z + 9$

$37 = 2z$

$\frac{37}{2} = \frac{2z}{2}$

$18.5 = z$

5-2

KeyConcept Equivalent Proportions

Symbols	The following proportions are equivalent. $\frac{a}{b} = \frac{c}{d}$, $\frac{b}{a} = \frac{d}{c}$, $\frac{a}{c} = \frac{b}{d}$, $\frac{c}{a} = \frac{d}{b}$
Examples	$\frac{28}{50} = \frac{x}{755}$, $\frac{50}{28} = \frac{755}{x}$, $\frac{28}{x} = \frac{50}{755}$, $\frac{x}{28} = \frac{755}{50}$.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} \ldots$$